Rendezvous of Heterogeneous Robots in Minimum Time - Theory and Experiments

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ABSTRACT

The task of multiple, physically-separated mobile robots meeting at a point is considered in this paper. This task, termed as *rendezvous*, is studied when the mobile robots have *unequal speeds*. An algorithm for computing the minimum time rendezvous point (denoted by P^*) for a pair of robots moving amidst polygonal obstacles is presented. The algorithm is based on the notion of visibility graph and computes P^* exactly. Comparisons with an alternate approach based on level sets are given. An extension to rendezvous of three robots (in minimum time), based on the notion of Apollonius circles is also presented. An experimental setup consisting of multiple mobile robots, fabricated in-house, is described. Experiments with the robots confirm the efficacy of the proposed algorithms.

CCS CONCEPTS

• **Embedded and Cyber-Physical Systems –> Robotics, Distributed Artifical Intelligence –> Multi-Agent Systems, Control Methods –> Robotic Planning**;

KEYWORDS

Mobile Robotics, Robotic Autonomy, Multi-Agent Systems, Rendezvous, Minimum Time, Unequal Speeds, Obstacles

1 INTRODUCTION

Two or more autonomous robots coming together to perform a task cooperatively is of interest in a number of applications. For example, one mobile robot (equipped with an arm) may pick some item from a second mobile robot and transfer it to a third. The three robots may be in physically different locations to begin with. The task is therefore to first bring them together. This is termed as the *rendezvous problem* which we study in this paper.

The rendezvous problem has its origins in the early work on satellite rendezvous [1] and search games [2]. Considerable work has been done on this problem for mobile robots during the last decade. The authors in [3] present a distributed algorithm for converging autonomous (point) robots with limited visibility. The rendezvous problem is studied in [4] in the context of cyclic pursuit. An approach based on proximity graphs is presented in [5] while an algorithm for rendezvous of point robots in a simply-connected and non-convex environment with constraints on visibility sensors is presented in [6]. A strategy to achieve rendezvous without coordinates or communication between agents is presented in [7]. An approach based on level sets is presented in [8]. Some experiments in rendezvous are reported in [9]. Energy considerations during rendezvous are studied in [10]. An optimal control formulation for rendezvous of an unmanned aerial vehicle with an unmanned ground vehicle is presented in [11] while rendezvous in the context of electric sails is discussed in [12]. Other related works on coordinated motion include [13], [14], [15], [16], [17].

While several variants of the rendezvous problem have been studied in the past, one aspect that has not been adequately explored is *minimum time rendezvous of heterogeneous robots with constraints.* This is illustrated in Figure 1. The robot with an arm mounted moves with lower (average) speed than the other (labelled mobile base). The goal is to bring the two robots together. Further, the robots face constraints in the form of obstacles in the environment. We then pose the following question: *What is the smallest time (from start) the robots should travel before they meet ?*

We address this problem with the assumption that each robot can be represented as a point mass (similar to the assumptions in [3], [6]). We present an algorithm for determining the point in the plane that meets the minimum time requirement for a pair of robots moving with unequal speeds in the absence of obstacles. We then examine the scenario where obstacles are present in the environment. We assume that the space occupied by machines, furniture, etc. in an indoor environment can be represented by polygons of arbitrary shape (they could be nonconvex) and the location of (vertices of) the obstacles is known to every robot. We develop an algorithm for meeting of a pair of robots, with unequal speeds, in minimum time amidst these obstacles. We show that the computation of minimum time location for a pair of robots amidst polygonal obstacles takes no more than the time required to compute the shortest path from one robot to another.

The theory is then extended to rendezvous of three robots in minimum time. Simulations of the algorithms developed are presented. Comparison of the proposed algorithm for two robots amidst obstacles with an alternate approach based on level sets [8] is presented. It is observed that the proposed algorithm returns the exact minimum time point amidst obstacles. An experimental setup consisting of custom-fabricated mobile robots is then described. Experiments with the robots are presented and they confirm the ability to rendezvous without any communication amongst the robots.

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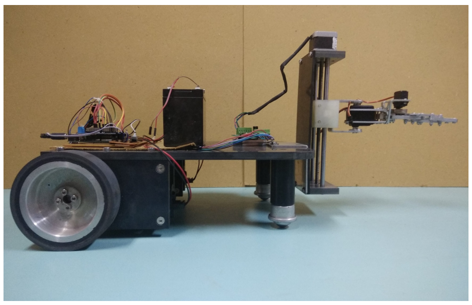
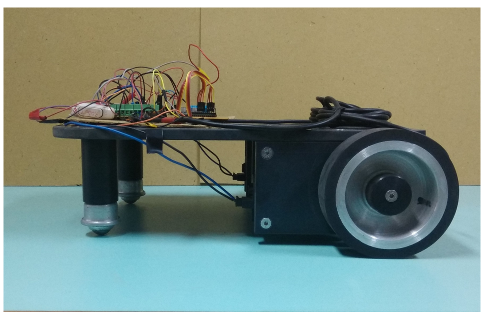
The remainder of this paper is organized as follows. Section 2 presents the definitions of terms used in the remainder of the paper. Section 3 describes the proposed algorithms for minimum time rendezvous of a pair of robots. Extensions to three robots are presented in section 4. Section 5 presents simulations and comparisons. Experiments are presented in 6. Section 7 concludes the paper.



(a) Robot with Arm Mounted



(b) Mobile Base

Figure 1: Heterogeneous Robots Attempting Rendezvous

2 DEFINITIONS AND TERMINOLOGY

We assume distinct initial locations in the plane for the robots and further the robots, in general, have unequal speeds. Given any point P in the plane, the time taken by a robot R to arrive at P will depend on the Euclidean distance to P (from R) as well as the speed of R. We therefore present a definition (Definition 2.1) that is required for describing what is meant by minimum-time rendezvous.

Definition 2.1. The *time for rendezvous* of two or more robots (denoted by t_p) at a given location, P, is defined as the total time taken by the last robot that arrives at P.

In other words, the time t_p is computed as the maximum of all times taken by the (various) robots to arrive at P. Gathering at an arbitrary point P is not advantageous from the point of view of energy consumption and therefore we need a point that is 'optimal' in some sense. This leads to identifying a point P^* as given by Definition 2.2.

Definition 2.2. P^* corresponds to a location in the plane, that, in comparison to any arbitrary location P, has a lower time for rendezvous: $t_{p^*} < t_p$, for all P.

We note that P^* is unique and minimizes the maximum of travel times from initial locations of all the robots. The minimum time itself is denoted by t_{p^*}.

The notion of visibility graph is required in developing algorithms to compute P^* particularly in the presence of polygonal obstacles. We define this next.

Definition 2.3. Two points, x and y, are (mutually) visible if the line segment xy does not intersect the interior of any object. This notion of visibility of a point pair is used to define the complete visibility graph of an environment consisting of a set of n polygonal objects. The complete visibility graph is a non-directed graph G specified as follows: (i) The nodes of G are the vertices of the n objects. (ii) Two nodes of G are connected by a line segment if and only if either the segment joining them is an edge of one of the polygons, or if it lies entirely in the free space except possibly at its two endpoints.

The concept of Apollonius circle [18] is valuable for multiple robots attempting rendezvous. Definition 2.4 captures this.

Definition 2.4. The locus of points where two robots (travelling with different speeds) arrive simultaneously turns out to be a circle enclosing the robot location with lower speed. This circle is known as the Apollonius circle while the robot locations are referred to as its foci.

3 MINIMUM TIME RENDEZVOUS OF A PAIR OF ROBOTS

Let the two mobile robots be located at A and B and let their average speeds be given by V_a and V_b respectively. We first present a result (given by Proposition 3.1) that identifies the region of the plane containing P^*. We then discuss about narrowing down the search to locate P^*, in the absence of obstacles, using V_a and V_b.

PROPOSITION 3.1. *The minimum time rendezvous point P^* lies on the line segment AB.*

Proof: The time taken for rendezvous is directly proportional to the distance between A and B. This time is minimized if the distance between A and B is the least. The length of the line segment AB corresponds to the least (Euclidean) distance between A and B. As a result, the search for P^* can be limited to the line segment AB. **Q.E.D.**

3.1 Minimum Time Rendezvous of Two robots In The Absence of Obstacles

From Definitions 2.1 and 2.2, we note that finding P^* corresponds to minimizing the maximum time travelled by the robots. The coordinates of P^* can be expressed in terms of the locations of the robots and their speeds as given by Lemma 3.2.

LEMMA 3.2. For two robots (whose locations are denoted by A and B), travelling with speeds V_a and V_b respectively, the minimum time rendezvous point P^* is given by (1).

$$P^* = \frac{(V_b \times A + V_a \times B)}{V_a + V_b} \quad (1)$$

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Proof: The time taken for rendezvous is the maximum of travel times of all robots prior to rendezvous (Definition 2.1) and P^* is the location that minimizes this maximum time. For an arbitrary location P (in AB), whose travel time from A is higher, an attempt to minimize this time brings P closer to A while the opposite occurs when minimizing the maximum time from B. Consequently, the minimum occurs when the travel times of both robots are identical, as given by (2).

$$\frac{AP^*}{V_a} = \frac{BP^*}{V_b} \tag{2}$$

As a result, P^* is computed as the point on AB that divides the line segment in the ratio $V_a : V_b$, as given by (1). **Q.E.D.**

An algorithm for minimum time rendezvous of a pair of robots in the absence of obstacles is straightforward and is directly based on Lemma 3.2. We now proceed to handle the case of an environment with static obstacles. As indicated earlier, the area occupied by furniture, equipment etc. can be represented by polygonal obstacles.

3.2 Minimum Time Rendezvous of a Pair of Robots Amidst Obstacles

In the presence of obstacles, the robot locations need not be visible (see also Definition 2.3) to each other. As a result, the search for the minimum time rendezvous point cannot be restricted to the line segment joining the robot locations A and B. Theorem 3.3 describes the location of P^* with respect to the robot and obstacle locations.

THEOREM 3.3. *Minimum time rendezvous point, P^*, lies on the shortest path from one robot (A) to the other (B).*

Proof: This is an extension of the result given by Proposition 3.1. When obstacles are present, the shortest path is not necessarily the segment joining A and B (since it may intersect the interior of one or more obstacles). The shortest path instead comprises of segments whose endpoints are either A, B, or the vertices of the polygons [19]. The minimum time rendezvous point, P^*, therefore lies on this path. **Q.E.D.**

3.3 Algorithm for Minimum time Rendezvous of Two robots Amidst Obstacles

The algorithm for minimum time rendezvous requires characterization of the shortest path in terms of visibility graph (Definition 2.3). This characterization is provided by Theorem 3.4. We note that A and B can also be thought of as point polygons with one vertex each [19].

THEOREM 3.4. *The shortest path is a subpath of the complete visibility graph formed from the vertices of the polygonal obstacles.*

Proof: The proof is a consequence of the definition of the complete visibility graph and Theorem 3.3. Clearly, the shortest path cannot pass through any point p in the interior of an obstacle. Further, any path outside the obstacles that does not pass through the vertices would have a length longer than one that is restricted to the obstacle vertices. **Q.E.D.**

The algorithm for minimum time rendezvous is therefore as follows.

Algorithm Min_Time_Rendezvous_Two_Robots
INPUT: Location of robots A, B, average speeds of the robots, namely V_a and V_b, and n polygonal obstacles with a total of m vertices.
OUTPUT: Minimum time rendezvous point P^*.
STEP 1: Construct the complete visibility graph of the environment.
STEP 2: Calculate the shortest path between A and B among n obstacles.
STEP 3: Starting from the location of one of the robots (namely, A), consider a line segment of length equal to that of the shortest path calculated in step 2 and denote the segment by R_iR_{temp} where R_i can corresponds to A while R_{temp} corresponds to a dummy (fictitious) location of the robot.
STEP 4: Use Eq. (1) on R_iR_{temp} to get a point P^*_{temp}. Denote the length of line segment from R_i to P^*_{temp} by L.
STEP 5: Traverse along the shortest path (computed in Step 2) by length L (obtained in Step 4) and identify two vertices (C_i and C_f), which appear on the path just before and after traversing by L.
STEP 6: Denote by d_i and d_f the distances from A to C_i and C_f and use Eq. (3) to mark off P^* on the portion of the shortest path between C_i and C_f.

$$P^* = \frac{(d_f - L) \times C_i + (L - d_i) \times C_f}{(d_f - d_i)} \tag{3}$$

REMARK 1. *Steps 2 to 5 of the algorithm are illustrated in Figure 2. In Step 3 of* **Algorithm Min_Time_Rendezvous_Two_Robots**, *one can start from B instead of A. In Step 5, C_i (or C_f) may correspond to the initial location of A (or B). Steps 4 and 5 in the algorithm can be expressed equivalently in terms of time travelled as the robot moves from A to obtain the minimum time point P^* and the corresponding time t_{p^*}.*

We assume that time overhead for turning of a robot at any corner (of an obstacle) is accommodated in the average speed of the robot.

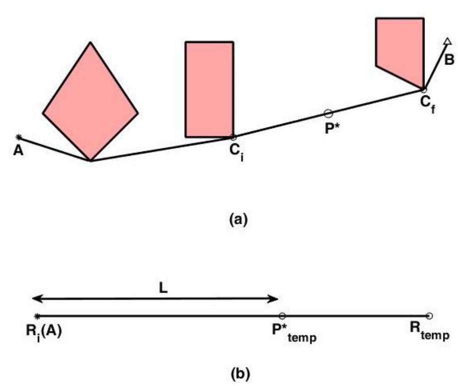

Figure 2: Illustration for Algorithm Min_Time_Rendezvous_Two_Robots

REMARK 2. **Algorithm Min_Time_Rendezvous_Two_Robots** *has a complexity of $O(m^2)$ since construction of the visibility graph and calculation of the shortest path dominate this. For an environment*

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with polygonal obstacles having a total of m vertices, the complete visibility graph followed by the shortest path can be obtained in $O(m^2)$ time [19].

4 MINIMUM TIME RENDEZVOUS OF THREE ROBOTS

The general approach to handle the minimum time problem for three or more robots is based on the notion of Apollonius circles (Definition 2.4). The main result leading to the development of an algorithm is given by Lemma 4.1.

LEMMA 4.1. *The minimum time rendezvous point (P^*) of three robots A, B and C, travelling with speeds V_a, V_b and V_c corresponds to one of the following: (i) The point of intersection of the three Apollonius circles constructed for the three pairs of robots with the lowest time for rendezvous or (ii) The three points obtained via Lemma 3.2 for every pair of robot locations.*

Proof: The proof for (i) is as follows. The minimum time rendezvous point (P^*) corresponds to a location where the three robots arrive simultaneously. This follows from the fact that any point adjacent to such a location would demand higher time of arrival for one of the robots, thus increasing the overall time for rendezvous. Since, an Apollonius circle is characterized by points that take equal time from two robots (Definition 2.4), the point of intersection of the three circles (one for each pair) corresponds to P^*. If the three circles intersect at multiple locations, point with the lowest time for rendezvous is P^*.

Suppose (i) does not hold (i.e., there is no common intersection point). P^* is then a function of only two robot locations and it can be computed via Eq. (1) in Lemma 3.2. This establishes part (ii) in the statement of the lemma. **Q.E.D.**

An algorithm for minimum time rendezvous of three robots in the absence of obstacles can be directly obtained from Lemma 4.1. We present this next.

Algorithm Three_Robots_Min_Time_Rendezvous
INPUT: Location of robots A, B and C with average speeds V_a, V_b and V_c respectively.
OUTPUT: Minimum time rendezvous point P^*.
STEP 1: Construct Apollonius circles for all the three pairs of robots and compute all the points of intersection. Denote by P^*, the point with lowest time for rendezvous. Proceed to Step 3. If no such intersection occurs, go to step 2.
STEP 2: Compute the three locations for the three pairs of robots as per Lemma 3.2 and the respective times for rendezvous. Denote the point with least time as P^*.
STEP 3: Output the minimum time rendezvous point, namely P^*.

Steps in **Algorithm Three_Robots_Min_Time_Rendezvous** are illustrated in Figure 3.

5 SIMULATIONS AND COMPARISONS

To assess the performance and accuracy of the proposed schemes, we have performed a simulation of the minimum time rendezvous algorithm for two robots amidst obstacles and compared with the scheme in [8].

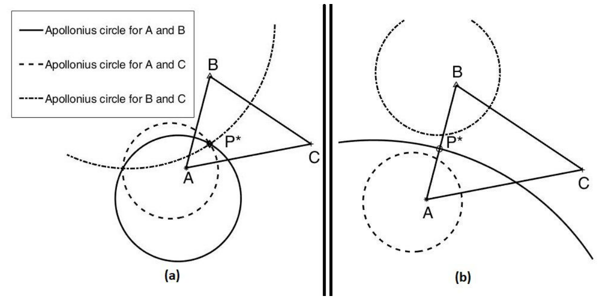

Figure 3: Illustration for Algorithm Three_Robots_Min_Time_Rendezvous; Case (a): All the three Apollonius circles intersect, Case (b): P^* is a function of only two robot locations

We first briefly describe the alternate approach to minimum time rendezvous presented in [8]. The authors in [8] address this problem for a group of heterogeneous robots that may include ground or air robots. The notion of level sets is introduced to solve this problem by first computing an arrival time map for each robot, subject to various constraints (on speed, dynamics etc.). The arrival time maps of the robots are then combined to locate the rendezvous point. In particular, the maximum of all the arrival time maps is computed at each point. The location with the smallest maximum is declared as the optimal rendezvous point.

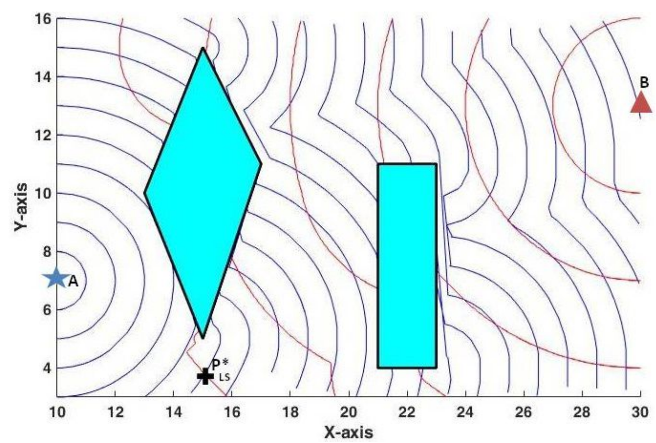

Figure 4: Simulation of minimum time rendezvous point using level set method for two robots A and B with obstacles

We have simulated the approach in [8] for a pair of ground (mobile) robots A (indicated by a blue star) and B (indicated by a red triangle) shown in Figure 4. A and B have average speeds of 1 and 3 units respectively. The coordinates of the obstacles and the robots are given in Table 1. The arrival time maps for robots A and B are constructed at intervals of one second. The global minimum (which corresponds to the minimum time rendezvous point) is denoted by P^*_{LS} (shown in the Figure 4). It is obtained by intersection of maps constructed for one second interval. The corresponding time for rendezvous is 6 seconds.

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Table 1: Coordinates of robots and obstacle vertices as well as P^* (due to proposed method) and P^*_{LS} (due to level set approach); Vertex labels are indicated in Figure 5

	A	B	O_{11}	O_{12}	O_{13}	O_{14}	O_{21}	O_{22}	O_{23}	O_{24}	P^*	P^*_{LS}
x	**10**	**30**	**15**	**17**	**15**	**13**	**21**	**23**	**23**	**21**	**15.27**	**15.05**
y	**7**	**13**	**5**	**11**	**15**	**10**	**4**	**4**	**11**	**11**	**5.27**	**3.75**

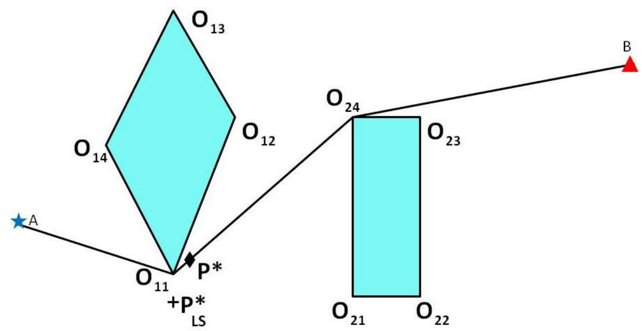


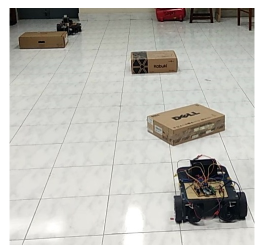
Figure 5: Simulation of minimum time rendezvous point using proposed algorithm for two robots A and B with obstacles

Figure 5 shows the minimum time rendezvous point, denoted by P^*, computed by the proposed algorithm for the same pair of robots A and B and the same environment. The time required for rendezvous is calculated to be 5.77 seconds. The observations based on the simulations of the two methods ([8] and proposed) are as follows. The level set method is computationally intensive although it is capable of leading to a solution that is correct and complete under resolution. The method proposed in this paper gives the exact location of the minimum time rendezvous point using only a few selected points in the environment, (namely vertices of obstacles) for the computation. Hence, unlike the level set method, the proposed method is computationally efficient as well as implementable on hardware platforms such as those based on microcontrollers.

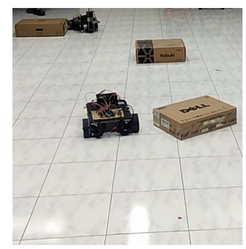
6 EXPERIMENTAL RESULTS

The proposed algorithms have also been implemented on custom-fabricated mobile robots. Each robot is equipped with an Arduino Uno (with ATMEGA328P-PU microcontroller), rotary encoder, battery and Pololu VNH5019 dual channel DC motor driver. The DC motors used have a maximum speed of 21 RPM. The computations are performed by the microcontroller on the Arduino Uno. The Arduino Uno interacts with the Pololu motor driver via an Adafruit PCA9685 16-Channel controller which generates the required PWM signals for the Pololu driver. The flexibility of Adafruit PCA9685 with respect to the large number of channels (each supporting a 12-bit resolution individual PWM controller) is taken advantage of in the design.

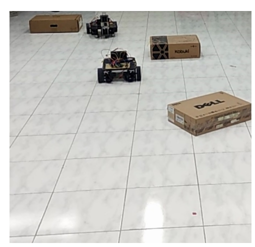
The first experiment involves the robot with arm mounted (A) and mobile base (B) with average speeds of 400 cm/min and 800 cm/min respectively. Three obstacles are placed in between as shown in Figure 6. Since the obstacle locations are known in advance, the visibility graph for the obstacles is computed off-line. When the initial location of robots ($A \equiv (30, 18)$, $B \equiv (0, 9)$) is provided, the visibility graph is updated. The minimum time rendezvous point is computed to be at (19.88,15.04). Four instants in the experiment are provided in the snapshots in Figure 6. The approach does not require any communication between the robots to achieve rendezvous.



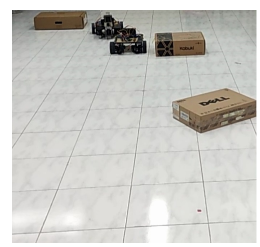
(a) Initial position of robots



(b) Turning of robots from imidiate obstacles



(c) Intermediate position of robots on the shortest path



(d) Final position of robots achieving rendezvous

Figure 6: Minimum time rendezvous of two robots with unequal velocities amidst obstacles.

The second experiment involves three robots, A (robot with arm mounted), B (mobile base) and C (mobile base), with average speeds of 400 cm/min, 600 cm/min and 800 cm/min respectively. Figure 7 shows snapshots of experiment performed. As before, a distributed approach without any communication between robots is followed. The task of solving equations relating to the intersection of Apollonius circles is performed on the Arduino Uno.

7 CONCLUSION

We have considered the problem of minimum time rendezvous of heterogeneous mobile robots travelling at unequal speeds in this paper. Algorithms for (i) minimum time rendezvous amidst obstacles for a pair of robots and (ii) minimum time rendezvous for three robots are presented. Simulations of the algorithms have been

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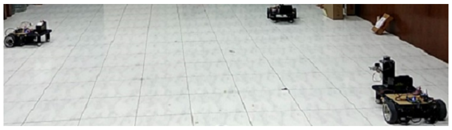
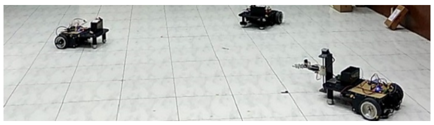
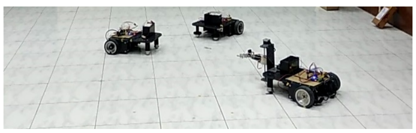


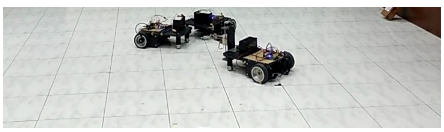
(a) Initial position of robots



(b) Robots taking turn



(c) Intermediate position of robots



(d) Final position of robots achieving rendezvous

Figure 7: Minimum time rendezvous of three robots with unequal velocities.

performed. It is observed that the proposed algorithm for a pair of robots amidst obstacles computes the minimum time rendezvous point exactly. Comparison with an alternate approach based on level sets is also described. Experiments with custom-fabricated mobile robots have also been performed to verify the effectiveness of the proposed schemes.

The proposed algorithm for minimum time rendezvous of three robots moving with unequal speeds does not readily extend to an environment with obstacles since it is not clear how the notion of Apollonius circles can be extended to this setting. It would be of interest to study the minimum time rendezvous problem for larger number of robots amidst obstacles.

ACKNOWLEDGMENTS

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